4. Magnetic Resonance Imaging(1)

Lectures 17, 18

Medical Imaging Systems

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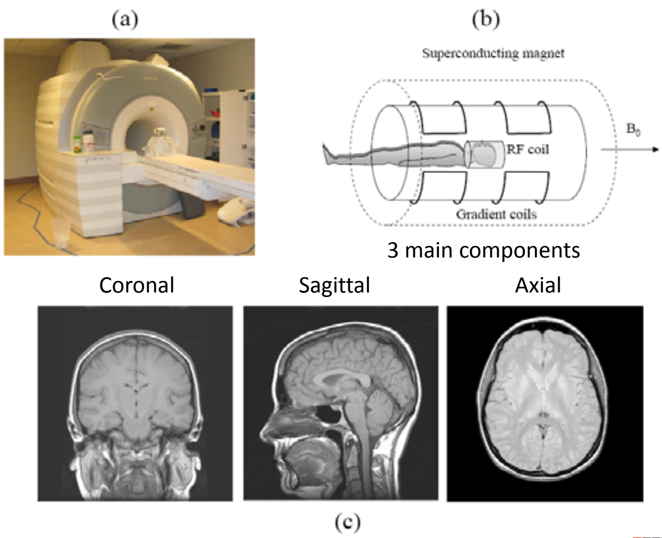
Contents

1. Introduction
2. Effects of a strong magnetic field on protons in the body
3. Effects of radiofrequency pulse on magnetization
4. Faraday induction: the basis of MR signal detection
5. T1 and T2 relaxation times
6. Signals from lipid
7. The free induction decay

Introduction

- The first MRI image: Paul Lauterbur in 1973 → won Nobel prize for medicine in 2003 with Peter Mansfield
- Advantages
  - Non-ionizing radiation
  - Images can be acquired in any 2-3D plane
  - Excellent soft-tissue contrast
  - Good spatial resolution (< 1mm)
  - Can provide information other than anatomy such as blood flow and water diffusion
- Disadvantages
  - Acquisition is much slower than CT/Ultrasound, comparable to PET
  - Need to exclude patients who has metallic implants
  - Much expensive than CT or ultrasound units

Introduction



Video Lectures

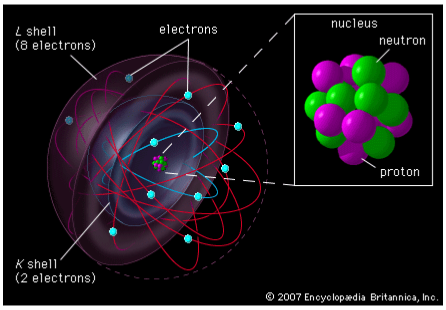
- http://youtu.be/7aRKAXD4dAg
- http://youtu.be/jUKdVBpCLHM
- http://youtu.be/GjLvu1hOAAA

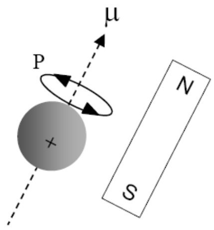
Magnetic field effects on protons

- A typical value of magnetic field, B_0, is 3T (30,000Gauss)
- Examples of magnetic field value
  - Earth: 30μT at equator, 70μT at poles
  - Refrigerator: 10mT
  - Clinical MRI: 1.5~3T
  - Research NMR: 4.7~11T +
- Quantum mechanical description
- Classical description

Quantum Mechanical Description

- A nuclei that has odd number of protons, neutrons, or both spins around an internal axis of rotation with a given value of angular momentum (P) or called 'spin'.
- Hydrogen atom is one of them (a single proton)
- This angular momentum produces a magnetic moment (μ)
- Now, this nuclei acts like a small bar magnet





Quantum Mechanical Description

- The magnitude of angular momentum of the proton is quantized (i.e. it can only have discrete values and spin quantum number, *I*, is ½ for a hydrogen).

$$\left|\vec{P}\right| = \frac{h}{2\pi}[I(I+1)]^{1/2}$$

- Therefore, $\left|\vec{P}\right| = \frac{h}{2\pi}\frac{\sqrt{3}}{2}$ for a hydrogen
- The proton's magnetic moment is proportional to the magnitude of angular momentum

$$|\vec{\mu}| = \gamma\left|\vec{P}\right|$$

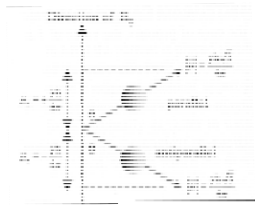
where γ is gyromagnetic ratio (42.6 MHz/Tesla for protons)

- Therefore, the magnetic moment also has a single, fixed value

$$|\vec{\mu}| = \frac{\gamma h}{2\pi}[I(I+1)]^{1/2}$$

Quantum Mechanical Description

- Therefore, $|\vec{\mu}| = \frac{\gamma h \sqrt{3}}{4\pi}$ for a hydrogen
- With a strong magnetic field, μ will have only the values given by $\mu_z = \gamma P_z = \frac{\gamma h}{2\pi} m_I$
  where m_I is the nuclear magnetic quantum number, and can take values, I, I-1, …. -I , For hydrogen, I is ½ and -½
- Therefore, $\mu_z = \pm \frac{\gamma h}{4\pi}$ and this makes a proton rotates with an angle of 54.7° ($cos\theta = \frac{\frac{\gamma h}{4\pi}}{\frac{\gamma h\sqrt{3}}{4\pi}} = \frac{1}{\sqrt{3}}$, $\theta = cos^{-1}\frac{1}{\sqrt{3}} = 54.74^o$)



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Quantum Mechanical Description

- The energy of ith spin state (E_i) is directly proportional to the value of m_I (nuclear magnetic quantum number) and magnetic field strength B_0

$$E_i = -\mu_z B_0 = -m_I \frac{\gamma h B_0}{2\pi} = \mp \frac{\gamma h B_0}{4\pi} \text{ (for hydrogen)}$$

- The energy difference between anti-parallel and parallel (ΔE) is

$$\Delta E = \left|E_{-\frac{1}{2}} - E_{\frac{1}{2}}\right| = \left|\left[\left(\frac{1}{2}\right)\left(\frac{\gamma h B_0}{2\pi}\right)\right] - \left[-\left(\frac{1}{2}\right)\left(\frac{\gamma h B_0}{2\pi}\right)\right]\right| = \frac{\gamma h B_0}{2\pi}$$

where h is Plank's constant (6.63 X 10^{-34} J s)

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Quantum Mechanical Description

- The relative number of nuclei in each configuration can be obtained by using a Boltzmann equation.

$$\frac{N_{antiparallel}}{N_{parallel}} = \exp\left(-\frac{\Delta E}{kT}\right) = \exp\left(-\frac{\gamma h B_0}{2\pi kT}\right)$$

where k is the Boltzmann coefficient (1.38 X 10^{-23} J/K)

- A first-order approximation of $e^{-x} \approx 1 - x$
- Therefore, $\frac{N_{antiparallel}}{N_{parallel}} = 1 - \frac{\gamma h B_0}{2\pi kT}$ and

$N_{parallel} - N_{antiparallel} = N_{total} \frac{\gamma h B_0}{4\pi kT}$

where N_{total} is the total number of protons in the body

- In 1.5T and 1 million protons, only 5 protons difference between parallel and antiparallel orientation

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Quantum Mechanical Description

- We should note that if I=0, then there is no angular momentum and thus no magnetic moment → no MR signal

TABLE 4.1. Properties of Nuclei Found at High Abudance in the Body

Nucleus	Atomic Number	Atomic Mass	I	$\gamma/2\pi$(MHz/T)	MRI Signal
Proton	1	1	1/2	42.58	Yes
Phosphorus	15	31	1/2	17.24	Yes
Carbon	6	12	0	–	No
Oxygen	8	16	0	–	No
Sodium	11	23	3/2	11.26	Yes

- That's why we use ^{13}C and ^{17}O for MRI

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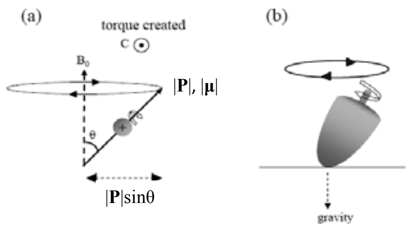
Classical Description

- We described nuclear magnetism using the quantum mechanical model, but it becomes harder to analyze the complicated MRI pulse sequences → use classical mechanics
- When protons are placed in the external magnetic field (B_o),
  1) the B_o field attempts to align the proton magnetic moment with itself
  2) This action creates a torque, C

$$\vec{C} = \vec{\mu} \times \overrightarrow{B_0} = i_N |\mu||B_0| sin\theta$$

where i_N is a unit vector normal to both $\vec{\mu}$ and $\overrightarrow{B_o}$

Classical Description

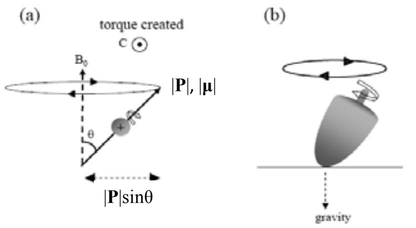


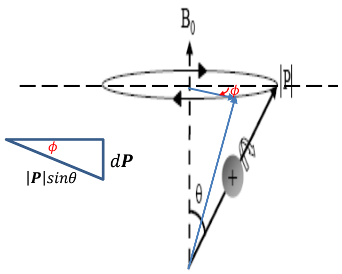
- In order to calculate the proton precession frequency, consider that the torque is defined as follows;

$$\vec{C} = \frac{d\vec{P}}{dt} = \vec{\mu} \times \overrightarrow{B_0}$$

- The magnitude of angular momentum perpendicular to $\boldsymbol{B}_0$ is $|\boldsymbol{P}|\sin\theta$
- In a short time dt, the magnetic moment precesses through an angle $d\phi$ producing a change $d\boldsymbol{P}$ in the angular momentum

Classical Description





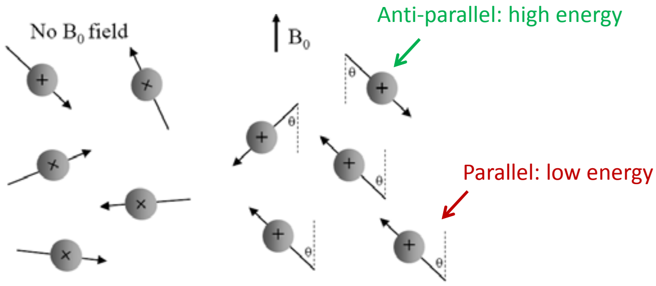
- Then, $\sin(d\phi) = \frac{d\boldsymbol{P}}{|\boldsymbol{P}|sin\theta} = \frac{\boldsymbol{C}dt}{|\boldsymbol{P}|sin\theta}$
- If $d\phi$ is small, then $\sin(d\phi) \sim d\phi$
- The angular precession frequency ω is given by $d\phi/dt$
- Therefore,

  $\omega = \frac{d\phi}{dt} = \frac{\boldsymbol{C}dt}{dt|\boldsymbol{P}|sin\theta} = \frac{\mu\times\boldsymbol{B}_0}{|\boldsymbol{P}|sin\theta} = \frac{\gamma\boldsymbol{P}\times\boldsymbol{B}_0}{|\boldsymbol{P}|sin\theta} = \frac{\gamma|\boldsymbol{P}||\boldsymbol{B}_0|sin\theta}{|\boldsymbol{P}|sin\theta} = \gamma B_0$

  where B_0 represent the magnitude of $\boldsymbol{B}_0$

B_o Field Effect

- For the normal state, the orientation of magnetic moment of protons are random→ net magnetic moment is zero
- If these protons are placed in the external magnetic field (B_0), they start to "precess" around the axis of B_0 with an angle of 54.7° → Larmor frequency (ω) =γB_0

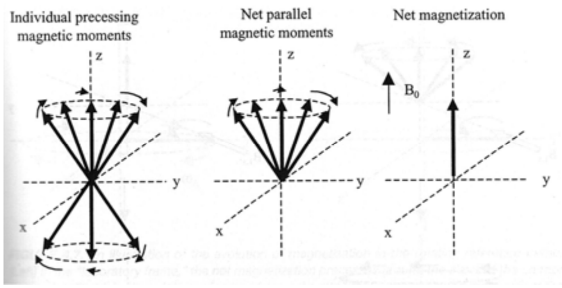


B_o Field Effect

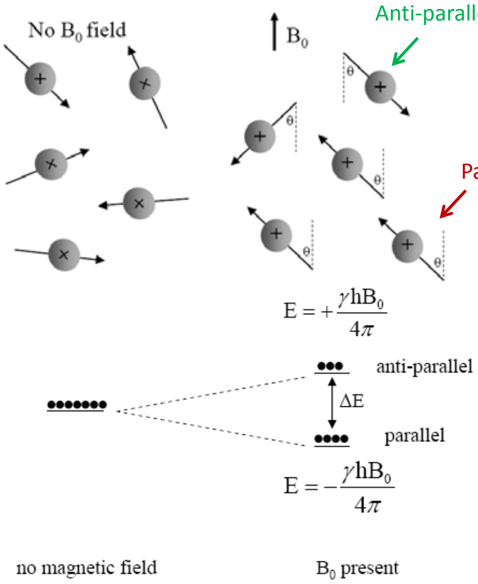
- Net magnetization of a sample is defined as M_o:

$$M_0 = \sum_{n=1}^{N_{total}} \mu_{Z,n} = \frac{\gamma h}{2\pi}\left(N_{parallel} - N_{antiparallel}\right) = \frac{\gamma^2 h^2 B_0 N_{total}}{2\pi^2 kT}$$

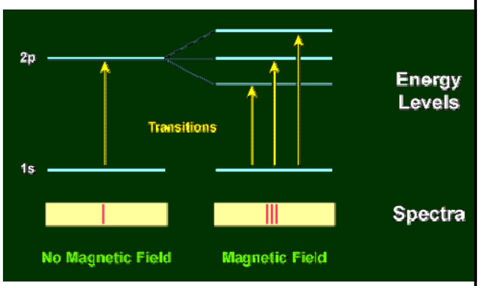
- To have a larger M_o,
  → higher γ, higher B_o, lower T



Proton energy levels



Zeeman effect

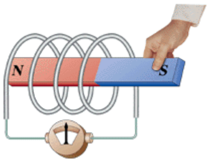


The number of split levels in the magnetic field is 2 * L + 1 (L: spin quantum number)

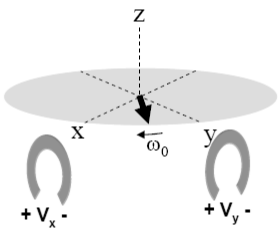
Faraday induction

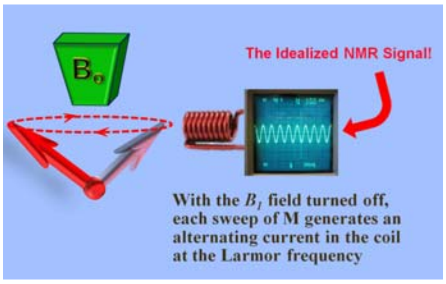
- Faraday's law of induction: the voltage (V) is induced in each of the loop with a value proportional to the time rate of change of the magnetic flux dφ:

$V = -n\frac{d\varphi}{dt}$ n: number of coils



$V_y \propto M_0\omega_0 sin\omega_o t,$
$V_x \propto -M_0\omega_0 cos\omega_o t$





RF pulse effects on magnetization

- From the previous slide, we can see that only oscillating magnetic field can be observed
- However, M_Z is not oscillating
- Therefore, we need to rotate precessing protons from *z* axis into *xy* plane
- Solution: apply a second magnetic field at a 90º angle to the z axis → creates a second torque → rotates the magnetization toward the *xy* plane
- This second magnetic field can be generated from a RF pulse

RF pulse effects on magnetization

- Energy levels of protons in magnetic field are analogous to
  - Energy levels in semiconductors
  - Vibrational and rotational energy levels in infrared spectroscopy
- To obtain an MR signal, energy must be supplied with a specific value ΔE to stimulate transitions between energy levels
- De Broglie's relationship of frequency: ΔE=hf:

$$hf=\Delta E=\frac{\gamma h B_o}{2\pi},\ f=\frac{\gamma B_o}{2\pi},\ \omega = \gamma B_o \text{ (Larmor frequency)}$$

→ *Resonance frequency*

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RF pulse effects on magnetization

- Transverse magnetization creation
  - In MRI, energy is applied as a short RF pulse 90° to B_o (creates B_1 field)
  - B_1 field produces a torque which causes the net magnetization to rotated towards the *xy*-plane
  - The tip angle (α) can be obtained by

$$\alpha = \gamma B_1 \tau_{B1}$$

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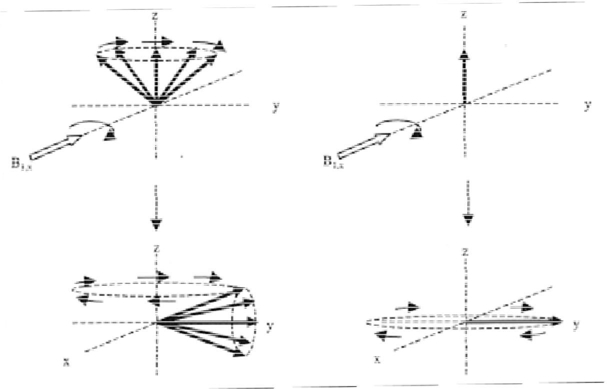
RF pulse effects on magnetization



- (A) Application of B_1 field along the *x* axis rotates the individual proton magnetic moments around the *x* axis toward *y* axis, (B) After applying the B_1 field for a certain time duration, the "cone" of magnetic moments has been rotated by 90°. The magnetic moments continue to precess around the B_0 axis, (C) and (D) show the vector model representation of (A) and (B)

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MR signal intensity

- From $V_y \propto M_0\omega_0 sin\omega_o t$, $V_x \propto M_0\omega_0 cos\omega_o t$, the size of MR signal is depending on
  1) The number of protons ;$M_0 = \frac{\gamma^2 h^2 B_0 N_{total}}{2\pi^2 kT}$
  2) The value of B_0; $M_0 = \frac{\gamma^2 h^2 B_0 N_{total}}{2\pi^2 kT}$
  3) The precession frequency, ω_0 (→ value of B_0)

- Overall, the image SNR $\propto B_0^{3/2}$

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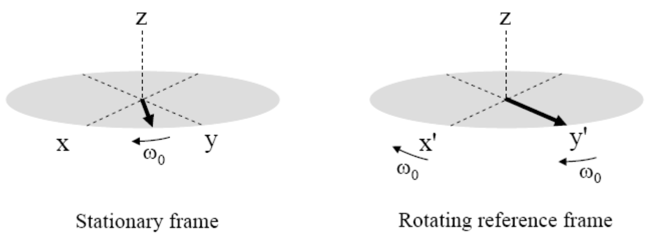
The rotating reference frame

- To visualize the proton precession, the concept of a 'rotating reference frame (x', y', z)' is useful

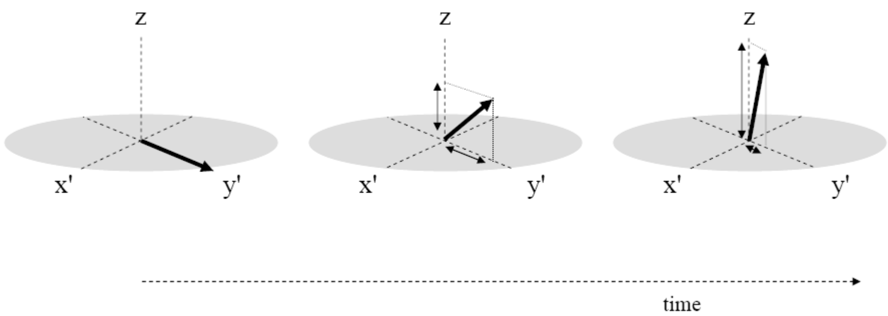


T1 relaxation times(spin-lattice)

$$M_z(t) = M_0 cos\alpha + (M_0 - M_0 cos\alpha)\left(1 - e^{-\frac{t}{T_1}}\right)$$

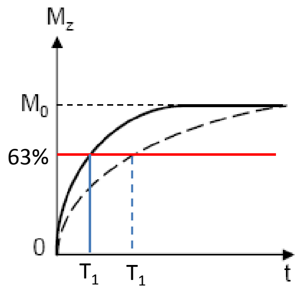
- After a 90º pulse,

$$M_z(t) = M_0\left(1 - e^{-\frac{t}{T_1}}\right)$$



T1 relaxation times

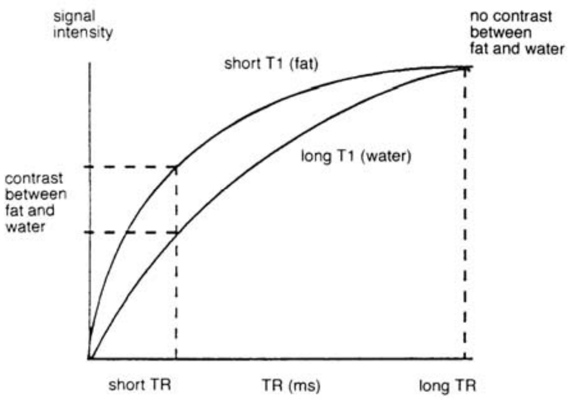
- The recovery of M_z magnetization after a 90° pulse



- When $t=5T_1$, M_z~99%M_0 , $t=3T_1$, M_z~95%M_0

T1 Weighting

- TR controls the amount of T1 weighting
- For T1 weighting the TR must be short

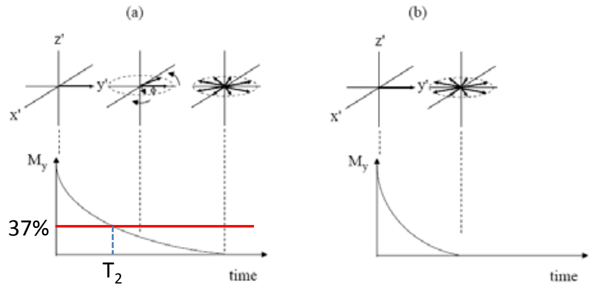


T2 relaxation times(spin-spin)

- If RF pulse is applied along the x-axis, M_y at time t after RF pulse is :

$$M_y(t) = M_0 \cdot sin\alpha \cdot e^{-\frac{t}{T_2}}$$

In practice, molecular dynamics (interactions with neighboring nuclei) causes a small spread in the precessional frequencies although all protons are assumed to precess at the same frequency



T2 relaxation times

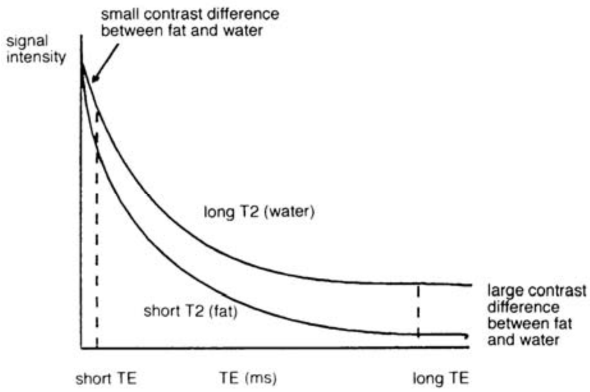
- Additional factor that leads M_x and M_y back to zero
  - Spatial inhomogeneity's of B_0 field
    1) Impossible to design a magnet which produces a perfect uniform B_0
    2) Local variations in magnetic field due to the different magnetic susceptibilities of different parts of the body (eg. air/tissue, bone/tissue boundaries)
  - T_2^+: the effects of local B_0 field inhomogeneity

$$\frac{1}{T_2^*} = \frac{1}{T_2^+} + \frac{1}{T_2}$$

  - In high resolution NMR, $T_2^* \cong T_2$ since T_2^+ is very small (sample is small and M_z is spatially homogeneous)

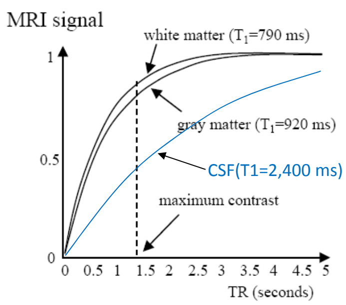
T2 Weighting

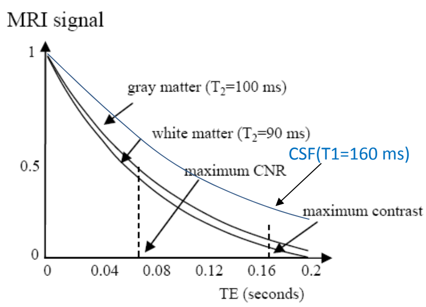
- TE controls the amount of T2 weighting
- For T2 weighting the TE must be long



Brain T_1 and T_2

- MRI signal is shown with 1.5T



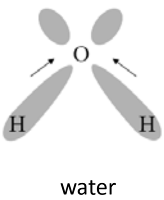


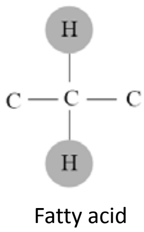
Video Tutorials

- 90 degree pulse and T1 relaxation
- http://youtu.be/P0YuwCphcRU
- T2 dephasing
- http://youtu.be/lKp67IqQjH4
- T2 dephasing topview
- http://youtu.be/_7oZMA0OuK4

Signals from lipid

- The majority of protons in lipid are in the form of –CH_2- groups in long-chain fatty acids
- The electron density distribution (shaded area) surrounding protons in water and lipid are shown in figures below
- Proton in water is less shielded due to strong electronegativity of oxygen than carbon in lipid





Signals from lipid

- An electron is a negatively charged particle, so produces a small magnetic field opposite in polarity to the B_0

$$B_{eff}=B_0(1-\sigma)$$

where σ is a shielding constant

- Therefore, the resonant frequency of the proton is

$$\omega = \gamma B_{eff} = \gamma B_0 (1-\sigma)$$

- σ for water is less than that for lipid, ($\sigma_{water} < \sigma_{lipid}$) therefore, water has a higher resonant frequency ($B_{eff\_water} > B_{eff\_lipid}$) (at 3T the difference is about 2500 rad/s or 400Hz

The free induction decay

- The signal precesses freely after the RF pulse has been turned off
- This M_x and M_y shows beat patterns which come from the two different resonant frequencies of lipid and water