3. Ultrasound Imaging(1)

Lectures 11, 12

Medical Imaging Systems

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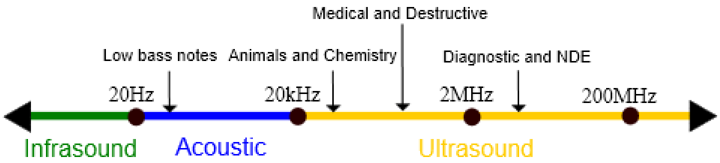
Introduction

- Infrasound: <20Hz, audible sound:20~20KHz, ultrasound: > 20KHz
- Ultrasound in clinical use (1~15MHz)



http://upload.wikimedia.org/wikipedia/commons/7/74/Ultrasound_range_diagram.svg

http://www.impactlab.net/2010/11/12/novel-metamaterial-vastly-improves-quality-of-ultrasound-imaging/

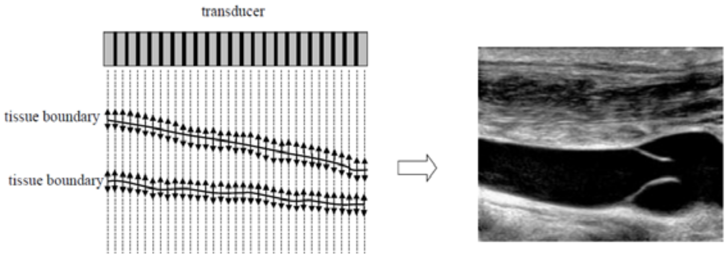
Introduction

- Ultrasound imaging is the least expensive and most portable among the standard clinical imaging techniques
- Real-time, continuous, safe (non-ionizing)
- Provide morphological and also blood flow information
- Used in obstetrics, gynecology, cardiovascular applications (measure fetal heart function, mitral valve), liver cysts,
- Guide the path and positioning of a needle in tissue biopsies
- Ultrasound in clinical use (1~15MHz)
- Speed of sound in tissue is ~1540 m/s (343 in air, 1484 in water, 5120 in iron)
- Ultrasound wavelengths in tissue is between ~0.1mm and 1.5mm ($\lambda = v/f$)

Abbe diffraction limit: $d = \frac{\lambda}{2n\sin\theta}$ *d*: spot radius, $n\sin\theta = NA$

Introduction

- Ultrasound is produced by transducer (~512 active elements)
- A transducer sends a series of pressure waves through the tissue
- A small fraction of the energy is backscattered at tissue boundaries toward transducer where it is detected
- Using the speed of sound in tissue, the depth of tissue boundary can be determined



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Wave propagation

- Ultrasound wave propagation is more like a movement of earthworm

http://youtu.be/oH8NMYi7qqw
http://youtu.be/REqxyVlT45M

Ultrasound Reference: https://www.nde-ed.org/EducationResources/CommunityCollege/Ultrasonics/Physics/wavepropagation.htm

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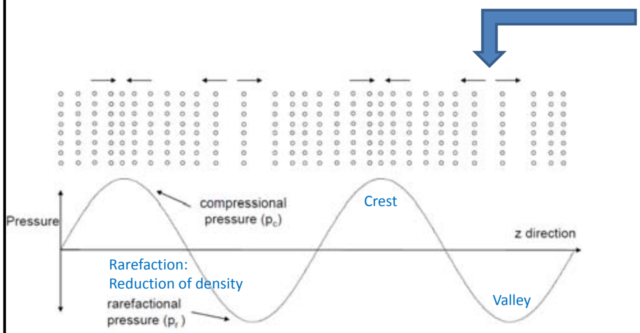
Wave propagation

- 3 dimensional lattice of small particles held together by elastic forces can be used to illustrate the principles of ultrasound propagation
- The effect of the ultrasound wave propagation on the displacement of the molecules within tissue is shown below



Individual particles oscillate as ultrasound passes

In ultrasound imaging the directions of vibration and wave propagation are the same
→ purely longitudinal wave

Wave propagation

- The value of particle displacement, W, is a few tenth of nanometer for typical ultrasound frequencies (1-15MHz) and related to c by

$$\frac{\partial^2 W}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 W}{\partial t^2} \text{ (wave equation)}$$

- The speed of the ultrasound wave in tissue, c, is

$$c = {}^{1}/_{\sqrt{\kappa\rho}}$$

  where ρ is tissue density and
  κ is compressibility ($\kappa = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T$)
  $1/\kappa$ is called the elastic modulus (in this case, K:bulk modulus)

- c is around 1540m/sec in most soft tissues

- More rigid ($\kappa \downarrow$)and less dense ($\rho \downarrow$) → higher the ultrasound propagation velocity

Wave propagation

- The particle velocity, u_z, along the propagation direction is

  $u_z = {}^{dW}/_{dt}$ where W is particle displacement
- u_z is 1-10cm/sec which is much lower than c
- Pressure (p) measured in pascals (1Pa=1Nm^{-2}=1kgms^{-2}m^{-2}=1kgm^{-1}s^{-2}) of the ultrasound wave at a particular point in the z-direction is

$$p = \rho c u_z$$

- Positive pressure: compressional forces
- Negative pressure: rarefactional forces

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Wave propagation

- Since the source is undergoing the sinusoidal motion, p(t) and u_z(t) can be expressed as

$$p(t) = p_0 e^{j\omega t}$$
$$u_z(t) = u_0 e^{j\omega t}$$

  where p_0 and u_0 are the peak pressure and particle velocity, respectively.
- The intensity i of the ultrasound wave is defined as the amount of power carried by the wave per unit area, and can be expressed as the product of p(t) and u_z(t).
- The average intensity I (watts/m^2) can be obtained by integrating i(t) over the period T of one cycle of the ultrasound wave:

$$I = \frac{1}{T}\int_0^T p(t)u_z(t)\,dt = \frac{1}{2}p_0 u_0$$

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Characteristic acoustic impedance

- Characteristic acoustic impedance (Z) of tissue is

$$Z = {}^{p}/_{u_z}$$

this can be considered as a direct analogue to Ohm's law (p: voltage, u_z: current)

- The value of Z is determined by the physical properties of the tissue

$$Z = {}^{p}/_{u_z} = {}^{\rho c u_z}/_{u_z} = \rho c = \rho \frac{1}{\sqrt{\rho\kappa}} = \sqrt{\frac{\rho}{\kappa}} \qquad \left(c = {}^{1}/_{\sqrt{\kappa\rho}}\right)$$

- Table 4.1 lists values of Z for tissues

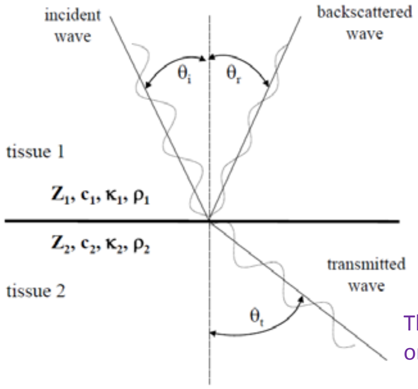
Table 4.1: Acoustic properties of biological tissues

	$Z \times 10^5$ ($g\ cm^{-2}\ s^{-1}$)	Speed of sound ($m\ s^{-1}$)	Density (gm^{-3})	Compressibility x10^{11} ($cm\ g^{-1}\ s^2$)
Air (in lungs)	0.00043	330	1.3	70 000
Blood	1.59	1570	1060	4.0
Bone	7.8	4000	1908	0.3
Fat	1.38	1450	925	5.0
Brain	1.58	1540	1025	4.2
Muscle	1.7	1590	1075	3.7
Liver	1.65	1570	1050	3.9
Kidney	1.62	1560	1040	4.0

The unit of compressibility is same as inverse of pressure

Wave reflection, refraction

- When an ultrasound wave encounters a boundary between two tissues with different Z values, a certain fraction of the wave energy is backscattered (reflected) towards the transducer→ source of ultrasound imaging



This boundary is assumed to be flat and much greater than ultrasound wavelength (eg. >>1mm for a 1.5MHz)

This refracted beam can cause displacement or an incorrect shape of organs

Wave reflection, refraction

- P_r: reflected pressure, P_t: transmitted pressure
- I_i: incident intensity I_r: reflected intensity, I_t: transmitted intensity
- R_p: reflection pressure coefficient, T_p: transmission pressure coefficient
- R_I: reflection intensity coefficient, T_I: transmission intensity coefficient

$$\theta_i = \theta_r, \quad \frac{sin\theta_i}{sin\theta_t} = \frac{c_1}{c_2} \text{ (Snell's law)}$$

$$R_p = \frac{P_r}{P_i} = \frac{Z_2 cos\theta_i - Z_1 cos\theta_t}{Z_2 cos\theta_i + Z_1 cos\theta_t}, \ T_p = \frac{P_t}{P_i} = \frac{2Z_2 cos\theta_i}{Z_2 cos\theta_i + Z_1 cos\theta_t}$$

$$R_I = \frac{I_r}{I_i} = \frac{(Z_2 cos\theta_i - Z_1 cos\theta_t)^2}{(Z_2 cos\theta_i + Z_1 cos\theta_t)^2}, \ T_I = \frac{I_t}{I_i} = \frac{4Z_2 Z_1 cos^2\theta_i}{(Z_2 cos\theta_i + Z_1 cos\theta_t)^2}$$

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Wave reflection, refraction

Incidence angle: 0 degree							
	Z1	Z2	C1	C2	Theta1(rad)	Theta2(rad)	R_I
air to muscle	0.00043	1.7	330	1590	0	0	0.998989
muscle to bone	1.7	7.8	1590	4000	0	0	0.412299
fat to muscle	1.38	1.7	1450	1590	0	0	0.010794
Incidence angle: 10 degree							
	Z1	Z2	C1	C2	Theta1	Theta2	R_I
air to muscle	0.00043	1.7	330	1590	0.174533	0.836668	0.999312
muscle to bone	1.7	7.8	1590	4000	0.174533	0.436851	0.443469
fat to muscle	1.38	1.7	1450	1590	0.174533	0.190414	0.011098
Incidence angle: 20 degree							
	Z1	Z2	C1	C2	Theta1	Theta2	R_I
air to muscle	0.00043	1.7	330	1590	0.349066	1.647915	1.000083
muscle to bone	1.7	7.8	1590	4000	0.349066	0.860428	0.543529
fat to muscle	1.38	1.7	1450	1590	0.349066	0.375043	0.011829
Incidence angle: 30 degree							
	Z1	Z2	C1	C2	Theta1	Theta2	R_I
air to muscle	0.00043	1.7	330	1590	0.523599	2.409091	1.000869
muscle to bone	1.7	7.8	1590	4000	0.523599	1.257862	0.733062
fat to muscle	1.38	1.7	1450	1590	0.523599	0.548276	0.012352

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Wave reflection, refraction

- The strongest reflected signal will be when the angle between the incident wave and the boundary is 90^o
- When the incidence angle is 0^o, the previous equations can be reduced to:

$$R_p = \frac{P_r}{P_i} = \frac{Z_2 - Z_1}{Z_2 + Z_1}, \; T_p = \frac{P_t}{P_i} = \frac{2Z_2}{Z_2 + Z_1}$$

$$R_I = \frac{I_r}{I_i} = R_p^2 = \frac{(Z_2 - Z_1)^2}{(Z_2 + Z_1)^2}, \; T_I = \frac{I_r}{I_i} = \frac{4Z_2 Z_1}{(Z_2 + Z_1)^2}$$

- The values of the reflectance and transmission pressure coefficients are $T_p = R_p + 1$
- Intensity of reflection and transmission coefficients is related by $T_I + R_I = 1$ (conservation of energy)

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Wave reflection, refraction

- The backscattered signal detected by transducer is maximized if the value of either Z_1 or Z_2 is zero
- But, it means the ultrasound beam will not reach deep inside body (eg. GI tract imaging if there is a pocket of air)
- Most cases of soft tissues, the intensity of the reflected wave is less than 0.1% of that of the incident wave
- Three cases from R_p, T_p, R_I and T_I

  1) $Z_1 >> Z_2$
     Ex) from tissue into air
     $T_p=0$, $T_I=0$, $R_I=1$ and $R_p=-1$, the negative sign of R_p signifies that the backscattered pressure wave undergoes a 180^o phase shift at the point that it encounters the boundary

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Wave reflection, refraction

- Three cases from R_p, T_p, R_I and T_I
  2) $Z_1 \sim Z_2$
     Ex) liver/kidney interface
     $T_p \sim 1$, $R_p < 1$, $T_I \sim 1$, $R_I << 1$.
     most of signals transmit trough the boundary and reach the deep tissue
  3) $Z_1 << Z_2$
     Ex) from tissue into bone
     $T_I=0$, $R_I=1$, $R_p=1$ and $T_p=2$
     $T_p=2$ means that the pressure at a single point at the boundary is actually twice that of the incident wave. As in case 1, almost all the energy is reflected back towards the transducer, except that there is no phase shift

Percent Reflection of US Energy

TABLE 3.2: PERCENT REFLECTION OF ULTRASONIC ENERGY FOR NORMAL INCIDENCE AT VARIOUS BOUNDARIES

Boundary	% reflection
Muscle/fat	1
Kidney/fat	0.6
Bone/muscle	41
Bone/fat	49
Soft tissue/air	99.9
Soft tissue/water	0.2

Scattering by small structures

- Rayleigh scattering: scatterer's size is much smaller than the wavelength ($I \propto 1/\lambda^4$)

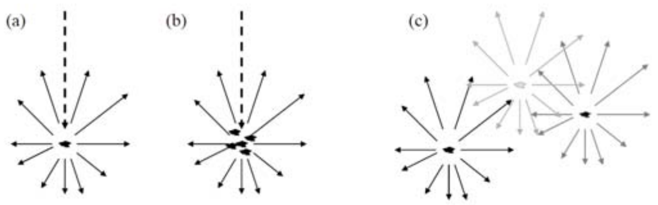
  ex) red blood cells (5~10μm)causes very strong Rayleigh scattering by ultrasound (0.1~1.5mm wavelength) → basis of Doppler ultrasound

Scattering by small structures

Rayleigh scattering

Constructive scattering by several structures which are close together

Constructive or destructive scattering by several structures which are relatively far from each other: Speckle→ considered to be a 'noise' in ultrasound imaging



(d)

An example of (c) case

Absorption of ultrasound energy

- Reflection, scatter, and absorption causes an attenuation of ultrasound energy in tissue
- Therefore, the signals received from deep tissue boundaries are much weaker than those from tissue boundaries close to the tissue surface
- Absorption converts ultrasound energy into heat
- There are two mechanisms of absorption takes place in biological tissue

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Classical absorption

- This is caused by friction between particles as they are displaced during the ultrasound wave passage
- This loss is characterized by an absorption coefficient, β_{class}, which is proportional to the square of the operating frequency

$$\beta_{class} = Af^2$$

where A is a constant about quantities of viscosity, thermal conductivity and f is ultrasound frequency
- The overall absorption is combination of relaxation and classical absorption, but in biological tissues at clinical frequencies, the relaxation absorption is dominant

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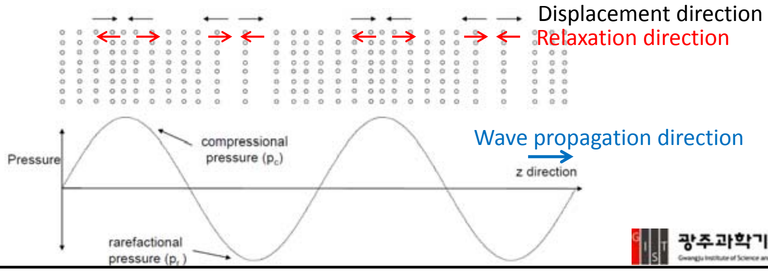
Relaxation absorption

- Different tissues have different elastic properties, which can be quantified by a relaxation time (τ)
- Relaxation time: the time that structures within the tissue require to return to their equilibrium position after having been displaced by the ultrasound wave
- There are two cases of a force applied to particles either constructive or destructive depending on the relaxation time and ultrasound propagation speed



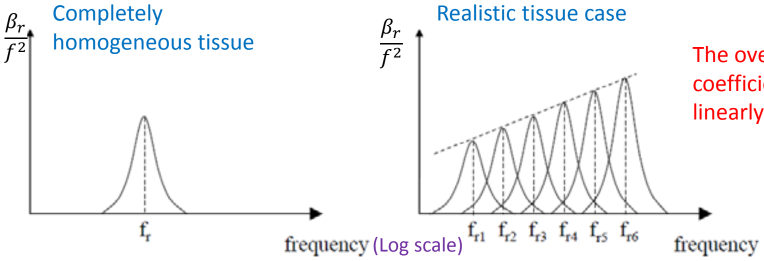
Relaxation absorption

- The relaxation absorption coefficient, β_r, is

$$\beta_r = \frac{B_0 f^2}{1+(f/f_r)^2}$$

$$\beta_{r,tissue} = \sum_n \frac{B_0 f^2}{1+(f/f_{r,n})^2}$$

- f_r is a relaxation frequency (=1/τ) and graphs of β_r as a function of frequency is shown below



Attenuation coefficients

- Attenuation of the ultrasound beam in tissue is the sum of absorption and scattering from small structures
- Both pressure and intensity of the ultrasound wave as a function of its propagation distance, z, are
  $I(z) = I(z=0)e^{-\mu z}$, $p(z) = p(z=0)e^{-\alpha z}$
  where μ is intensity attenuation coefficient and α is pressure attenuation coefficient (cm^{-1})
- μ is twice the value of α (μ=2α) and often stated in units of decibels (dB) per cm
  $\log_{10} e = 0.4343$
  $1\mu(dBcm^{-1}) = -\frac{1}{z} 10 log_{10} \frac{I(z)}{I(Z=0)} = 4.343\ \mu(cm^{-1}) \leftarrow I(z)=I(z=0)*\exp(-\mu z)$
  $1\alpha(dBcm^{-1}) = -\frac{1}{z} 20 log_{10} \frac{p(z)}{p(Z=0)} = 8.686\ \mu(cm^{-1}) = 1\mu(dBcm^{-1})$
- Rule-of-Thumb: 3dB reduction→ (1.995=~2) factor reduction in intensity (6dB→ 4 times, 9dB → 8 times reduction in intensity)

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Frequency Dependency of μ

- The frequency dependency of μ for soft tissue is $1dBcm^{-1} MHz^{-1}$ i.e. at 2MHz the attenuation coefficient is $2dBcm^{-1}$.
- For fat, it is approximately by $0.7f^{1.5}$ dB
- For air and bone are much higher, $45dBcm^{-1}MHz^{-1}$ and $8.7dBcm^{-1}MHz^{-1}$, respectively
- Ex) the intensity of 3MHz US beam entering tissue is $10mW/cm^2$. What will be the intensity of the beam at 4cm depth? (attenuation coefficient=1dB/cm·MHz)
- Sol) 3MHz→3dB/cm, 4cm→12dB, 12dB→16times intensity loss. Therefore, the intensity of the beam is $0.625mW/cm^2$

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